

# Can Planck-scale physics be seen in the cosmic microwave background ?

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We investigate the potential of observations of anisotropies in the cosmic microwave background (CMB) and large-scale structure in the Universe to detect possible modifications of standard inflationary models by physics beyond the Planck scale. A generic model of the primordial density fluctuations is investigated, and we derive constraints on its parameters from current data. We conclude that the currently available data do not put very stringent constraints on this model. Furthermore, we use simulated power spectra from the Sloan Digital Sky Survey (SDSS) and the Planck satellite to show that it is unlikely that a trans-Planckian signature of this type can be detected in CMB and large-scale structure data.

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## I. INTRODUCTION

Quantum gravitational effects are expected to modify the spectrum of primordial density fluctuations produced during the inflationary phase in the very early Universe. Since the primordial fluctuations are the seeds for the anisotropies in the cosmic microwave background (CMB) and for the large-scale structures we observe in the Universe, cosmological observations have the potential to shed light on Planck-scale physics. However, although enormous progress has been made in observational cosmology in the past few years, it is a highly non-trivial task to separate the primordial density fluctuations from the present-day, processed power spectra which we can measure. In this paper we show that the currently available data do not impose any stringent constraints on trans-Planckian modulations of the primordial power spectrum. Furthermore, we investigate whether future, more precise measurements will improve the prospects for detecting a signature for quantum gravity. Based on suggestions for the form and size of trans-Planckian effects on the primordial power spectrum (see e.g. [1, 2, 3, 4, 5, 6, 7]), we consider a model where a power-law spectrum is modulated by an oscillating function of the comoving wavenumber  $k$  [5, 6, 7]. By performing a full likelihood analysis of simulated data sets, we are led to the conclusion that it will be extremely difficult to detect trans-Planckian effects in CMB and large-scale structure data.

The structure of this paper is as follows. In section II

we give a brief summary of the work on trans-Planckian effects on the primordial power spectrum and introduce our model. Section III contains a discussion of the cosmological data, with an emphasis on the unavoidable smearing of power by the window functions. Section IV describes our likelihood analysis and the results,

## II. TRANS-PLANCKIAN PHYSICS AND THE PRIMORDIAL POWER SPECTRUM

The standard calculation of the perturbations produced during the inflationary phase is based on flat-space quantum field theory, and the initial conditions for the inflaton  $\phi$  are imposed in the infinite past. Since the Universe is expanding, this means that any given Fourier mode  $k$  was infinitesimally small at that time. Since our expectation is that ‘new physics’ becomes important on length scales below the Planck length, the standard calculation is probably too naive, and we should expect corrections from quantum gravitational effects. We do not know what the correct fundamental theory on scales comparable to and smaller than the Planck length is, and hence it is impossible to predict these corrections with a high degree of certainty. One can only devise plausible scenarios based on informed guesses as to what the significant features of a theory of quantum gravity should be like. Many proposals have been made with a varying degree of optimism regarding possible observable signatures [1, 2, 3, 4, 5, 6, 7]. An important point is whether the leading-order correction to the primordial power spectrum is linear in  $H/\Lambda$ , where  $H$  is the Hubble constant during inflation and  $\Lambda$  is the energy scale where new physics enters. If the correction is of higher order than linear, it will certainly be undetectable, so we

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will assume that it is linear. With one specific choice of vacuum, it was shown in [5, 6] that the primordial power spectrum modified by trans-Planckian effects is given by

$$P(k; \epsilon, \xi) = P_0(k) \left\{ 1 - \xi \left( \frac{k}{k_n} \right)^{-\epsilon} \sin \left[ \frac{2}{\xi} \left( \frac{k}{k_n} \right)^\epsilon \right] \right\}, \quad (1)$$

where  $\epsilon$  is the standard inflationary slow-roll parameter related to the inflaton potential  $V(\phi)$  by  $\epsilon = (16\pi)^{-1} M_{\text{P}}^2 (V'/V)^2$ ,  $M_{\text{P}} = 1/\sqrt{8\pi G}$  is the reduced Planck mass,  $\xi \sim 4 \times 10^{-4} \sqrt{\epsilon}/\gamma$ ,  $\gamma = \Lambda/M_{\text{P}}$  parametrizes the scale where trans-Planckian effects enter, and  $P_0(k)$  is the primordial power spectrum predicted by standard inflationary theory, typically given by the scale-free form  $P_0(k) \propto k^{n_s-1}$ , where  $n_s$  is the scalar spectral index. Finally,  $k_n$  corresponds to the largest scales measurable in the CMB. The effect is seen to be a modulation of  $P_0(k)$ , and the natural question to ask is whether this is detectable.

Based on simulated data for the Planck satellite [8], it was concluded in [6] that the modulation should be detectable in the CMB data from this upcoming mission. However, this conclusion was based on a Fisher matrix analysis, not on a full likelihood analysis. In fact, as we will show, the full likelihood analysis reveals that the issue is more complicated. The likelihood oscillates in parameter space, and one can be misled if one looks only at its local behaviour.

### III. OBSERVATIONAL DATA

#### A. Cosmic microwave background

The CMB temperature fluctuations are conveniently described in terms of the spherical harmonics power spectrum

$$C_l \equiv \langle |a_{lm}|^2 \rangle, \quad (2)$$

where

$$\frac{\Delta T}{T}(\theta, \phi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi). \quad (3)$$

Since Thomson scattering polarizes light there are additional power spectra coming from the polarization anisotropies. The polarization can be divided into a curl-free ( $E$ ) and a curl ( $B$ ) component, yielding four independent power spectra:  $C_{T,l}$ ,  $C_{E,l}$ ,  $C_{B,l}$  and the temperature  $E$ -polarization cross-correlation  $C_{TE,l}$ .

The WMAP experiment have reported data on  $C_{T,l}$  and  $C_{TE,l}$ , as described in Ref. [9, 10, 11, 12, 13]

We have performed the likelihood analysis using the prescription given by the WMAP collaboration which includes the correlation between different  $C_l$ 's [9, 10, 11, 12, 13]. Foreground contamination has already been subtracted from their published data.

In parts of the data analysis we also add other CMB data from the compilation by Wang *et al.* [14] which includes data at high  $l$ . Altogether this data set has 28 data points.

#### B. Large scale structure

The 2dF Galaxy Redshift Survey (2dFGRS) [15] has measured the redshifts of more than 230 000 galaxies with a median redshift of  $z_m \approx 0.11$ . An initial estimate of the convolved, redshift-space power spectrum of the 2dFGRS has been determined [16] for a sample of 160 000 redshifts. On scales  $0.02 < k < 0.15 h \text{ Mpc}^{-1}$  the data are robust and the shape of the power spectrum is not affected by redshift-space or nonlinear effects, though the amplitude is increased by redshift-space distortions. A potential complication is the fact that the galaxy power spectrum may be biased with respect to the matter power spectrum, i.e. light does not trace mass exactly at all scales. This is often parametrised by introducing a bias factor

$$b^2(k) \equiv \frac{P_g(k)}{P_m(k)}, \quad (4)$$

where  $P_g(k)$  is the power spectrum of the galaxies, and  $P_m(k)$  is the matter power spectrum. However, we restrict our analysis of the 2dFGRS power spectrum to scales  $k < 0.15 h \text{ Mpc}^{-1}$  where the power spectrum is well described by linear theory. On these scales, two different analyses have demonstrated that the 2dFGRS power spectrum is consistent with linear, scale-independent bias [17, 18]. Thus, the shape of the galaxy power spectrum can be used straightforwardly to constrain the shape of the matter power spectrum. However, when looking for modulations or other features in the primordial power spectrum using the 2dFGRS, one should bear in mind that what is measured is the convolution of the true galaxy power spectrum with the 2dFGRS window function  $W$  [16],

$$P_{\text{conv}}(\mathbf{k}) \propto \int P_g(\mathbf{k} - \mathbf{q}) |W_k(\mathbf{q})|^2 d^3 q, \quad (5)$$

and it was found in [19] that this convolution washes out any features in the primordial power spectrum for  $k < 0.1 h \text{ Mpc}^{-1}$ . However, combining the 2dFGRS power spectrum with CMB data breaks parameter degeneracies that are present if each dataset is analysed separately, and therefore a combination of large-scale structure and CMB data gives tighter constraints on the primordial power spectrum than the CMB alone.

#### C. Likelihood analysis

For calculating the theoretical CMB and matter power spectra we use the publicly available CMBFAST package

[20]. As the set of cosmological parameters we choose  $\Omega_m$ , the matter density,  $\Omega_b$ , the baryon density,  $H_0$ , the Hubble parameter,  $\tau$ , the optical depth to reionization,  $Q$ , the normalization of the CMB power spectrum,  $b$ , the bias parameter, and finally  $\epsilon$  and  $\xi$ . We restrict the analysis to geometrically flat models  $\Omega_m + \Omega_\Lambda = 1$ .

In principle one might include even more parameters in the analysis, such as  $r$ , the tensor to scalar ratio of primordial fluctuations. However,  $r$  is most likely so close to zero that only future high precision experiments may be able to measure it. The same is true for other additional parameters. Small deviations from slow-roll during inflation can show up as a logarithmic correction to the simple power-law spectrum predicted by slow-roll. [21, 22, 23] or additional relativistic energy density [24, 25, 26, 27, 28, 29, 30, 31, 32] could be present. However, there is no evidence of any such effect in the present data and therefore we restrict the analysis to the “minimal” standard cosmological model.

In this full numerical likelihood analysis we use the free parameters discussed above with certain priors determined from cosmological observations other than CMB and LSS. In flat models the matter density is restricted by observations of Type Ia supernovae to be  $\Omega_m = 0.28 \pm 0.14$  [33]. The current estimated range for  $\Omega_b h^2$  from BBN is  $\Omega_b h^2 = 0.020 \pm 0.002$  [34], and finally the HST Hubble key project found the value of  $H_0$  to be  $72 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1}$  [35]. The marginalisation over parameters other than  $\epsilon$  and  $\xi$  was performed using a simulated annealing procedure [36].

#### IV. RESULTS

In this section we present the results of the likelihood analysis of our selection of current cosmological data sets. First of all, in figure 1 we show 68 % and 95 % exclusion limits in the  $\xi$ - $\epsilon$  parameter space (all other parameters have been marginalised over) for the WMAP TT and TE power spectra only. Figure 2 shows how the constraints change when the compilation of pre-WMAP CMB data from [14], and in figure 3 the 2dFGRS power spectrum data have been included as well.

From these figures it is clear that there is no preference in the data for an oscillating power spectrum, at any significant level. The  $\epsilon = \xi = 0$  point is always within the  $1\sigma$  allowed range.

For the CMB-only data the current excluded range is for  $\xi \gtrsim 0.1$  and  $0.1 \gtrsim \epsilon \gtrsim 1$ .  $\xi$  is a measure of the amplitude of oscillations, and the bound on this parameter comes from the fact that small amplitude oscillations are not visible, given the precision of current experiments. Since  $\epsilon$  roughly determines the period of the power spectrum oscillations it is clear why only a very limited range is excluded. For small values of  $\epsilon$  the power spectrum does not go through a whole oscillation within the visible range of  $k$ -values. On the other hand, for very large  $\epsilon$  the oscillation period becomes small compared to the

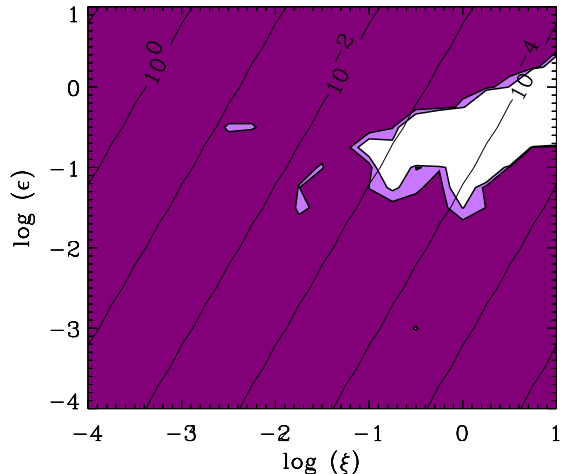


FIG. 1: 68% and 95% confidence exclusion plot of the parameters  $\epsilon$  and  $\xi$  for WMAP data only. The straight lines are isocontours for the parameter  $\gamma$ .

width of the power spectrum window function.

In the case where LSS data is included the conclusion is very similar, except for the presence of an excluded region beyond  $\xi = 1$  at low  $\epsilon$ . The reason for this simply is that when  $\xi > 1$  the power spectrum can take negative values for some  $k$ . Since the amplitude fitting of the spectrum is done by multiplying with a positive, real number this means that no good fit can possibly be obtained when starting from an underlying power spectrum with negative values. Of course it also reflects the transition to the unphysical region of parameter space since the power spectrum should be a positive definite quantity.

#### V. SIMULATED, HIGH-PRECISION DATA SETS

##### A. Simulated Planck data

We have simulated a data set from the future Planck mission using the following very simple prescription: We assume it to be cosmic variance (as opposed to foreground) limited up to some maximum  $l$ -value,  $l_{\text{max}}$ , which we take to be 2000. For the sake of simplicity we shall work only with the temperature power spectrum,  $C_{T,l}$ , in the present analysis. In fact the Planck detectors will be able to measure polarization as well as temperature anisotropies, but our simplification of using only  $C_{T,l}$  will not have a significant qualitative impact on our conclusions. For a cosmic variance limited full sky experiment the uncertainty in the measurement of a

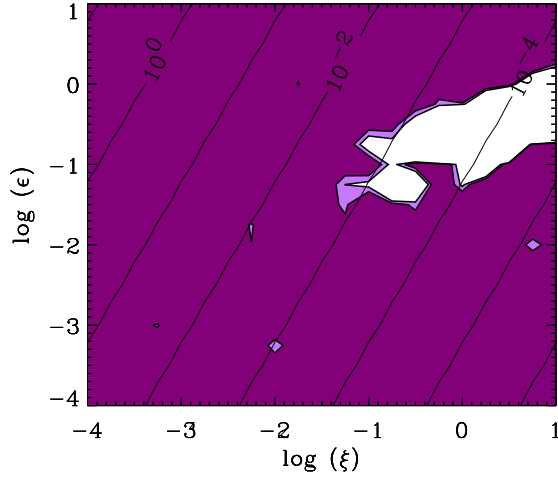


FIG. 2: 68% and 95% confidence exclusion plot of the parameters  $\epsilon$  and  $\xi$  for the WMAP data set combined with the pre-WMAP data compilation from Wang et al. The straight lines are isocontours for the parameter  $\gamma$ .

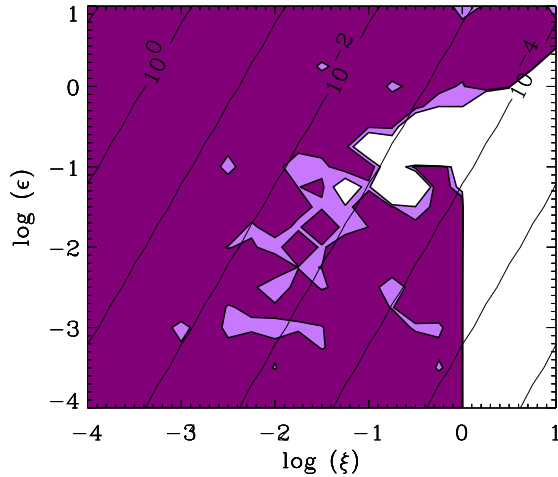


FIG. 3: 68% and 95% confidence exclusion plot of the parameters  $\epsilon$  and  $\xi$  with the previous CMB data plus additional data from the 2dF galaxy survey. The straight lines are isocontours for the parameter  $\gamma$ .

given  $C_l$  is simply

$$\frac{\sigma(C_l)}{C_l} = \sqrt{\frac{2}{2l+1}}. \quad (6)$$

It should be noted that taking the data to be cosmic variance limited corresponds to the best possible case. In reality it is likely that foreground effects will be significant, especially at high  $l$  (see Ref. [37]). Therefore, our estimate of the precision with which the oscillation pa-

rameters can be measured is probably on the optimistic side.

## B. Simulated SDSS data

The Sloan Digital Sky Survey (SDSS) [38] aims at measuring the redshift of approximately 1 million galaxies, and from this the galaxy power spectrum can be obtained with unprecedented accuracy. We will consider how this improves our ability to constrain trans-Planckian physics, assuming that the bias of galaxies with respect to dark matter is simple and scale-independent.

For a fixed survey strategy, the survey volume and the number density of galaxies in the redshift sample set a lower bound on the uncertainty in the estimated power spectrum. In the limit of a perfectly spherical volume-limited sample, the uncertainty in the estimated power per mode is roughly

$$\frac{\delta \bar{P}(k)}{\bar{P}(k)} \approx \sqrt{2 \frac{V_c}{V_k}} \left[ 1 + \frac{S(k)}{\bar{P}(k)} \right], \quad (7)$$

[39] where  $P(k)$  and  $\bar{P}(k)$  are the true and estimated power spectra,  $S(k) = 1/\bar{n}$  is the shot noise power for a mean galaxy density  $\bar{n}$  (we will use  $\bar{n} = 2 \times 10^{-3} h^3 \text{Mpc}^{-3}$ ),  $V_c = (2\pi)^3/V_S$  is the coherence volume in the Fourier domain for a survey with volume  $V_S$ , and it is assumed that we average the power estimates over a shell in Fourier space with volume  $V_k \approx 4\pi k^2 \Delta k$ . That is, we average the power over all angles, and over bins with  $\Delta k > 2\pi/R$ , where  $R$  is the survey depth, which we will take to be  $500 h \text{Mpc}^{-1}$ .

In constructing a mock window function we follow [39], so the window function in Fourier space is the Fourier transform of the product of the survey mask with the redshift selection function  $\varphi(r)$ . Since we are anyway interested in constructing an optimistic estimate of the window function, we assume that the survey geometry is simple and has full coverage of  $2\pi$  steradians, [40] and adopt a magnitude limit  $B_J < 18.9$ .

With these optimistic assumptions, the effect of the window function is less dramatic than in the case of the 2dFGRS. In figure 4 we show the ratio of the power spectrum with the standard, scale-invariant primordial  $P_0(k)$  to a modulated one with  $\epsilon = 0.03$ ,  $\xi = 0.023$ , before (solid line) and after (dashed line) convolution with our mock window function. As can be seen, the main effect is some smoothing of large-scale power, but the modulation is still clearly visible.

## C. The CMB window function

CMB data do not directly measure the underlying primordial power spectrum of fluctuations. Rather, they measure the spectrum folded with a transfer function in

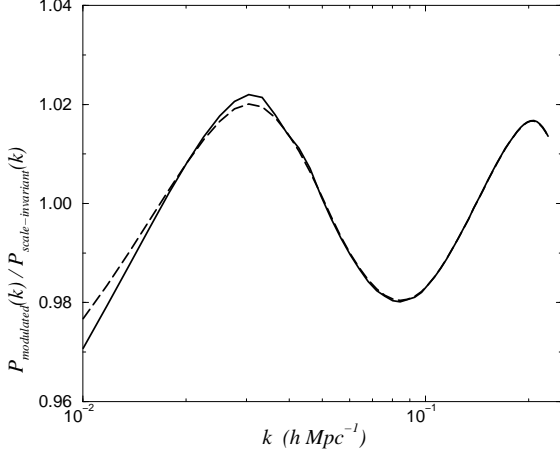


FIG. 4: Ratio of modulated to unmodulated power spectrum, before (solid line) and after (dashed line) convolution with the mock SDSS window function.

the following sense

$$C_l = \int \frac{dk}{k} P(k) \Delta_l^2(k), \quad (8)$$

where  $\Delta_l^2(k)$  is the transfer function taken at the present,  $\tau = \tau_0$ , where  $\tau$  is conformal time. Following the line-of-sight approach pioneered by Seljak and Zaldariagga [20] this transfer function can be written as

$$\Delta(k) = \int_0^{\tau_0} d\tau S(k, \tau) j_l[k(\tau - \tau_0)], \quad (9)$$

where  $S$  is a source function, calculated from the Boltzmann equation, and  $j_l(x)$  is a spherical Bessel function. However, in order to get a very rough idea about the effective window function  $w_l(k) = \Delta_l^2(k)/k$  of CMB we approximate  $S$  with a constant to obtain

$$w_l(k) \propto \begin{cases} \left(\frac{k\tau_0}{l}\right)^l \sim 0 & \text{for } k\tau_0 \lesssim l \\ \frac{1}{k^3} & \text{for } k\tau_0 \gtrsim l \end{cases} \quad (10)$$

This simple equation shows several things: (a) A feature at some specific wavenumber  $k = k_*$  has the greatest impact on the CMB spectrum at  $l_* \simeq k_*\tau_0$ . For a flat, matter dominated universe,  $\tau_0 = H_0/2$ , yielding  $l_* \simeq 2k_*/H_0$ . (b) The CMB windows function is quite broad, and narrow features in  $P(k)$  are accordingly difficult to detect.

Starting from the power spectrum given in Eq. (1) with  $P(k) = P_0(k)(1 + \Delta P(k))$  and

$$\Delta P(k) = \xi \left(\frac{k}{k_0}\right)^{-\epsilon} \sin \left[ \frac{2}{\xi} \left(\frac{k}{k_0}\right)^\epsilon \right] \quad (11)$$

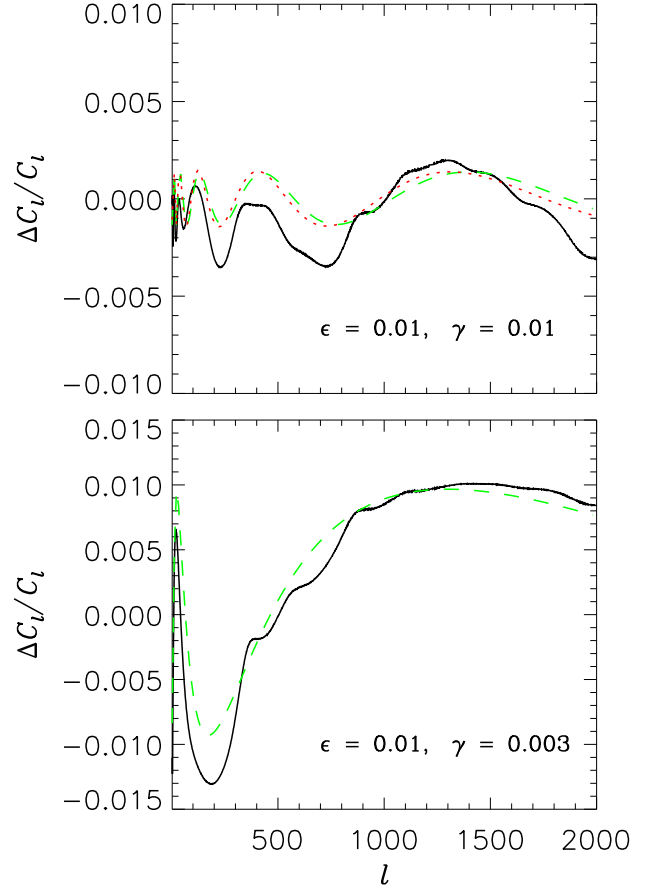


FIG. 5: Ratio of modulated to unmodulated power spectrum, both the exact result (solid line) and the approximation Eq. (16) (dashed line).

we can write the change in the CMB power spectrum,  $\Delta C_l$  as

$$\frac{\Delta C_l}{C_l} = \frac{\int_{l/\tau_0}^{\infty} \frac{\Delta P(k)}{k^3} dk}{\int_{l/\tau_0}^{\infty} \frac{P_0(k)}{k^3} dk} \quad (12)$$

Since  $k_0 \simeq l/\tau_0$  this can be recast in the relatively simple form

$$\frac{\Delta C_l}{C_l} = \frac{1}{2l^2} \int_l^{\infty} \xi q^{-\epsilon-3} \sin \left[ \frac{2}{\xi} q^\epsilon \right] dq. \quad (13)$$

In the limit where  $|\frac{2l^\epsilon \epsilon}{\xi(2+\epsilon)}| \gg 1$  one can use the approximation

$$\frac{\Delta C_l}{C_l} = \frac{\xi^2}{\epsilon} l^{-2\epsilon} \cos \left[ \frac{2l^\epsilon}{\xi} \right] \quad (14)$$

In figure 5 we show the quantity  $\Delta C_l / C_l$  for the same two sets of parameters as in Fig. 2 of Ref. [6], i.e. ( $\epsilon = 0.01, \gamma = 0.01$ ) and ( $\epsilon = 0.01, \gamma = 0.003$ ).

## D. Results

In figures 6 and 7 we show the result of a likelihood analysis for simulated Planck and SDSS data. The underlying model used was a standard concordance  $\Lambda$ CDM model ( $\Omega_m = 0.3$ ,  $\Omega_\Lambda = 0.7$ ,  $\Omega_b h^2 = 0.024$ ,  $n_s = 1$ , and  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ), with the addition of a modulated spectrum with parameters,  $\epsilon = \xi = 0.01$ . From the analysis of the simulated Planck data set we find that the best fit model for  $\epsilon = \xi = 0.01$  has  $\chi^2 = 2063.1$  for  $\nu = 1992$  degrees of freedom. This is roughly within the expected  $1\sigma$  interval of  $\chi^2 = \nu \pm \sqrt{2\nu} = 1992 \pm 63$ . However, the actual best fit point of the analysis was at  $\epsilon = 0.022, \xi = 0.053$  which has  $\chi^2 = 2046.5$ , seemingly ruling out the correct model at more than 99.9% confidence.

The problem with this analysis is that both models are decent fits within the expectations, but that the exact value of the likelihood function is extremely sensitive to  $\epsilon$  and  $\xi$ . This is partly due to the fact that the phase of the spectrum oscillation at a given  $k$ -value is strongly dependent on  $\epsilon$  and  $\xi$ . It should also be noted that any grid-based likelihood calculation is likely to fail when faced with this type of likelihood function, and that only stochastic algorithms like Markov Chain Monte Carlo (MCMC) [41] or simulated annealing [36] are likely to work.

If, instead of using the standard prescription of assigning likelihood according to  $\Delta\chi^2$  compared with the global best fit, one uses a robust estimation such as goodness-of-fit the plot becomes very different. Figure 8 shows the contours when taking the  $1\sigma$  region to be  $1992 \pm 63$ , i.e. the expected  $1\sigma$  interval for the  $\chi^2$  distribution. In this case the data is unable to discriminate the underlying model from one without any oscillation, but the huge number of non-connected regions in the likelihood fit disappear.

This shows very clearly that the likelihood estimator is in principle very powerful, but that it is not robust. Using a more robust estimator significantly reduces the sensitivity, but on the other hand also eliminates the possibility of unphysical biasing of the results.

From figure 8 it can also be seen that the exclusion region has the same topology as those using the present WMAP data (figures 1 and 2), and for the same reasons.

The analysis is almost equivalent when including the mock SDSS data. It should, however, be noted that the likelihood contours in figures 6 and 7 are very different, showing again the sensitivity of  $\Delta\chi^2$  to  $\epsilon$  and  $\xi$ . Figures 9 and 10 illustrate the degeneracy between models with widely different  $\epsilon$  and  $\xi$ .

## VI. DISCUSSION

We have performed a detailed analysis of how a logarithmic oscillation of the primordial fluctuation spectrum influences CMB and LSS observations. The motivation

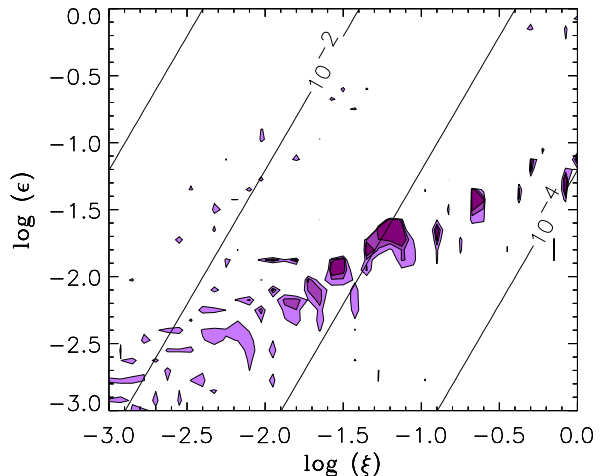


FIG. 6: 68% and 95% confidence exclusion plot of the parameters  $\epsilon$  and  $\xi$  for the simulated Planck data only. The straight lines are isocontours for the parameter  $\gamma$ .

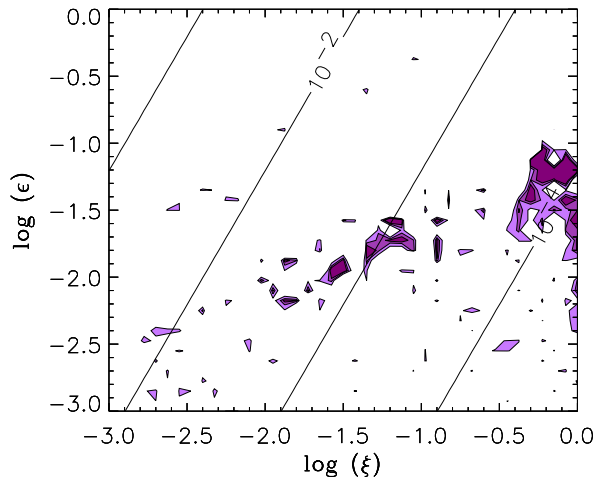


FIG. 7: 68% and 95% confidence exclusion plot of the parameters  $\epsilon$  and  $\xi$  for the simulated Planck data combined with the simulated SDSS data. The straight lines are isocontours for the parameter  $\gamma$ .

for this study was that trans-Planckian effects could lead to exactly such a modulation of the spectrum, but in fact our analysis of present and simulated future data sets is more general.

The analysis of present data shows that there is no evidence at a significant level for any oscillating behaviour of the power spectrum, but that only a very narrow region of parameter space is currently excluded. This region is not close to the expected values of the parameters  $\epsilon$  and  $\gamma$  for plausible models of Planck scale physics, such as

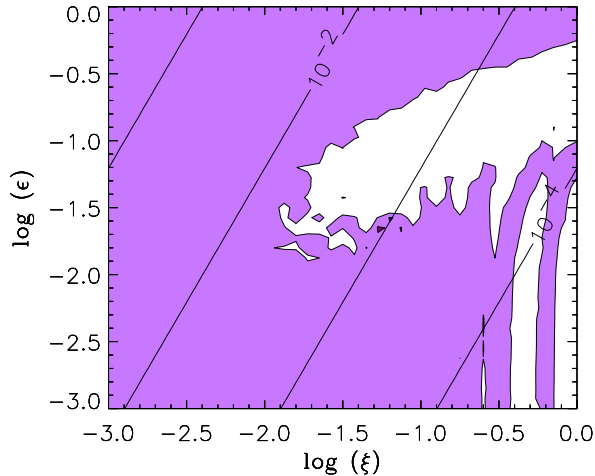


FIG. 8: 68% and 95% confidence exclusion plot of the parameters  $\epsilon$  and  $\xi$  when the  $1\sigma$  region is taken to be  $1992 \pm 63$ , the expected  $1\sigma$  interval for the  $\chi^2$  distribution. The straight lines are isocontours for the parameter  $\gamma$ .

the Horava-Witten model.

In order to study the possibility of detecting modulations with future data sets, such as those from the Planck satellite or the SDSS survey, we have constructed mock data sets and performed likelihood analyses on them. This analysis showed that the likelihood function is extremely sensitive to the parameters  $\epsilon$  and  $\xi$ , and that the likelihood function exhibits a large number of distinct local minima. This makes any analysis based on a  $\Delta\chi^2$  approach very difficult and any result is likely to be biased by unphysical effects. On the other hand, more robust methods, such as relying on a calculation of  $\chi^2$  alone without relying on  $\Delta\chi^2$  as compared to the global best fit, are much less sensitive. We have shown for one specific case, where parameters were chosen to be close to optimistic estimates of the Horava-Witten model, that using a standard likelihood analysis yields a biased parameter estimate, whereas a robust method is unable to distinguish the underlying model from one with no oscillations.

Our conclusion is that CMB and LSS data are in principle very sensitive to modulations in the underlying primordial power spectrum, but that in practise it is likely to be extremely difficult to make a positive detection of the small amplitude oscillations predicted, even with future high precision data.

### Acknowledgments

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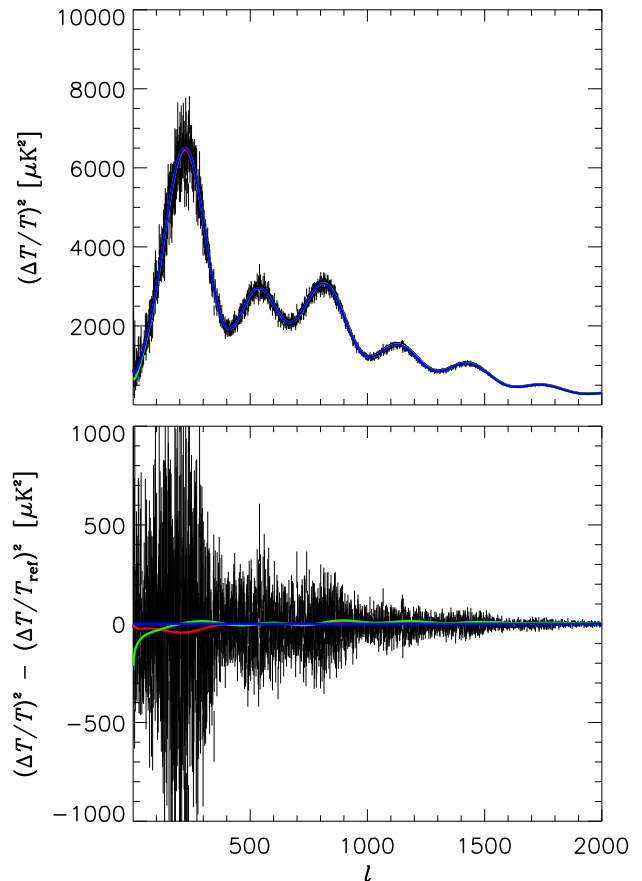


FIG. 9: The CMB power spectra for the best fits to the simulated data based on the reference model  $\epsilon = \xi = 0.01$  (blue line). The green line has  $\epsilon = 10^{-1.2}$ ,  $\xi = 10^{-0.3}$ , while the red line has  $\epsilon = 10^{-2.1}$ ,  $\xi = 10^{-1.95}$ . The bottom panel shows the residuals with respect to the reference model

for useful comments.



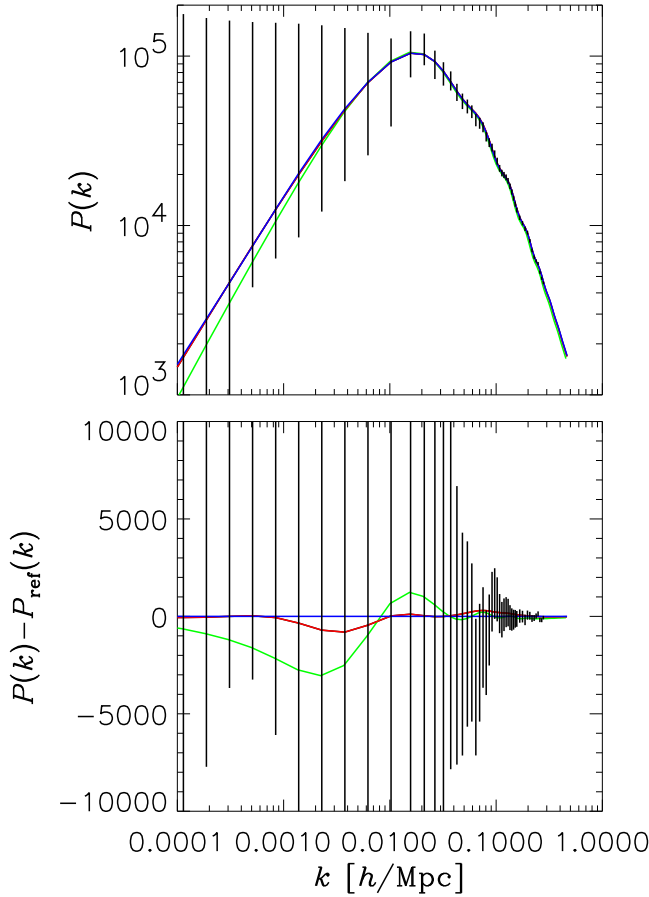


FIG. 10: The galaxy power spectra for the best fits to the simulated data based on the reference model  $\epsilon = \xi = 0.01$  (blue line). The green line has  $\epsilon = 10^{-1.2}$ ,  $\xi = 10^{-0.3}$ , while the red line has  $\epsilon = 10^{-2.1}$ ,  $\xi = 10^{-1.95}$ . The bottom panel shows the residuals with respect to the reference model.

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